On the Structure of the Electron

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Dedicated to Ibo

Usually the electron is described as a mathematical point with charge, mass, spin angular momentum, and electromagnetic field. Because of the unbounded energies this causes mathematical difficulties [1]. These can be avoided by considering a finite radius. For a "free electron at rest" a classical relativistic model is presented where an axisymmetric torus models the electron. This configuration "differentially rotates" around its axis of symmetry with superluminal speed.

Key words: Classical Field Theory; Relativistic Fluid Dynamics.

1. Description of the Model

If the electron is considered as a classical extended particle, the Coulomb repulsion force has to be compensated by nonelectromagntic forces. This is the main problem (see Rohrlich [2] and the references therein). The term "free electron at rest" in the abstract is used for a mathematical point, while the term "differential rotation" is related to a configuration of finite extent. In the latter the electron is described in terms of a model of continuum mechanics and vacuum electrodynamics, which means that ε and μ have the vacuum values ε_0 and μ_0, respectively. The equations of vacuum electrodynamics are written in SI-units:

$$\operatorname{div} \vec{B} = 0, \tag{1}$$

$$\operatorname{curl} \vec{E} + \partial_t \vec{B} = 0, \tag{2}$$

$$\operatorname{div} \vec{E} = \frac{q}{\varepsilon_0}, \tag{3}$$

$$\mu_0 \vec{j} = \operatorname{curl} \vec{B} - \frac{1}{c^2}\partial_t \vec{E}, \tag{4}$$

$$\vec{j} = q \vec{\upsilon}. \tag{5}$$

Here, q is the charge density, $c = (\varepsilon_0\mu_0)^{-\frac{1}{2}}$ is the vacuum speed of light, and the other symbols have their usual meaning.

Using the metric tensor

$$g_{\nu\mu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{6}$$

the 4-velocity and the gradient have the form

$$u^\nu = \gamma \begin{pmatrix} c \\ \upsilon_i, \end{pmatrix}, \quad \gamma = \left(1 - \frac{\upsilon^2}{c^2}\right)^{-\frac{1}{2}},$$
$$\partial_\nu = \begin{pmatrix} \frac{1}{c}\partial_t \\ \partial_i \end{pmatrix}, \quad \begin{matrix} \nu,\mu = 0,1,2,3, \\ i,j,k = 1,2,3 \end{matrix} \tag{7}$$

and the electromagnetic field is written as

$$F^{\nu\mu} = \begin{pmatrix} 0 & -\frac{E_j}{c} \\ \frac{E_i}{c} & -B_{ij} \end{pmatrix},$$
$$B_{ij} = \varepsilon_{ijk}B_k, \quad j_\mu = \begin{pmatrix} cq \\ -j_j \end{pmatrix}. \tag{8}$$

In continuum mechanics with electromagnetic force the simplest relativistic forms of mass and energy-momentum balance are

$$\partial_\nu \rho_0 u^\nu = 0, \tag{9}$$

$$\rho_0 u^\mu \partial_\mu u^\nu = F^{\nu\mu} j_\mu. \tag{10}$$

Here, the mass density $\rho_0 \geq 0$ is a Lorentz invariant which is not necessarily the rest mass density. Equations (9) and (10) yield in 3-vector notation

$$\partial_t \gamma\rho_0 + \operatorname{div} \gamma\rho_0\vec{\upsilon} = 0, \tag{11}$$

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$$c^2\gamma\rho_0\left[\partial_t\gamma + (\vec{v}\cdot\nabla)\,\gamma\right] = \vec{E}\cdot\vec{j}, \tag{12}$$

$$\gamma\rho_0\left[\partial_t\gamma\vec{v} + (\vec{v}\cdot\nabla)\,\gamma\vec{v}\right] = q\vec{E} + \vec{j}\times\vec{B}. \tag{13}$$

Here, (12) can be omitted because it is a consequence of (13). Note that (9) – (13) do not contain artificial non-electromagnetic forces as in [3 – 5] for instance.

Let us consider the time-independent (stationary) case. By using (3), (4), and (11) the momentum equation can be written in terms of the stress tensor as

$$\partial_j\left[\rho_0\gamma^2 v_i v_j - \varepsilon_0(E_iE_j - \frac{1}{2}E^2\delta_{ij}) - \frac{1}{\mu_0}(B_iB_j - \frac{1}{2}B^2\delta_{ij})\right]. \tag{14}$$

Now, multiply (14) by the position vector x_i, integrate over whole space, and assume that all fields tend to zero at infinity so that partial integration yields no boundary contributions at infinity. The result is then

$$\int(\rho_0\gamma^2 v^2 + \frac{\varepsilon_0}{2}E^2 + \frac{1}{2\mu_0}B^2)\mathrm{d}^3\tau = 0. \tag{15}$$

If the velocity field is such that the speed is everywhere smaller than the vacuum speed of light $(v < c)$, then γ is real and (15) admits only the trivial solution. This is called a "virial theorem", being the formal reason why in this simple case the Coulomb repulsion forces cannot be compensated by nonelectromagnetic forces. However, if one considers the superluminal case $v > c$, then $\gamma^2 < 0$ and one finds the following relations for the energies:

$$\int\rho_0\gamma^2 v^2\mathrm{d}^3\tau < U_M < 0, \tag{16}$$

where

$$U_\mathrm{M} = c^2\int\rho_0\gamma^2\mathrm{d}^3\tau \tag{17}$$

is the mechanical energy, whose density is the 0,0-component of the kinetic energy-momentum-tensor. Using (15) then yields the total energy

$$U = U_\mathrm{M} + U_\mathrm{E} + U_\mathrm{B} = c^2\int\rho_0\mathrm{d}^3\tau > 0, \tag{18}$$

where

$$U_\mathrm{E} = \frac{\varepsilon_0}{2}\int E^2\mathrm{d}^3\tau, \quad U_\mathrm{B} = \frac{1}{2\mu_0}\int B^2\mathrm{d}^3\tau \tag{19}$$

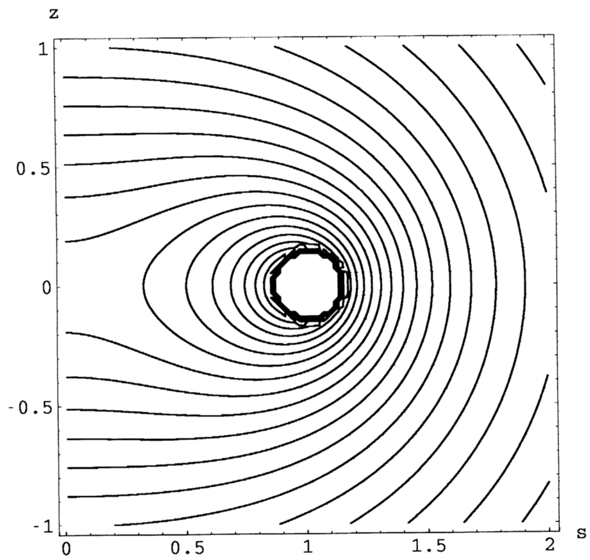

Fig. 1. Poloidal cut of the external equipotential surfaces $\Phi = \text{const}$.

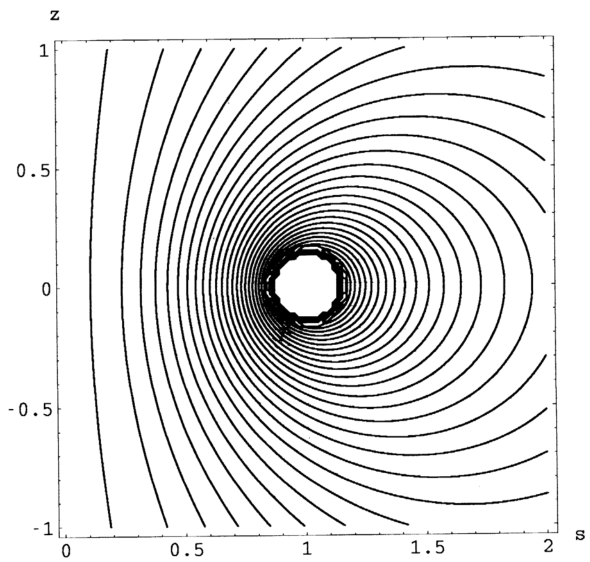

Fig. 2. External magnetic field lines $\psi = \text{const}$. in the poloidal plane.

are the electromagnetic energies. The energy expression (18) is important for the discussion of the stability problem.

Now, the following axisymmetric model is discussed. The electron of finite extent has the form of an axisymmetric torus of finite aspect ratio which differentially rotates around its axis of symmetry with superluminal speed $(v > c)$. The condition $v > c$ does not contradict the principles of special relativity, because it is not necessary to transmit signals inside the electron. Both the electromagnetic field and the flow are stationary and axisymmetric. The electromagnetic field is poloidal, the current density and the flow are toroidal, while the mass and charge densities are axisymmetric scalars.

Let s, ϕ, z be cylindrical coordinates. The time-independent axisymmetric electromagnetic field is described by the equations

$$\vec{E} = -\nabla\boldsymbol{\Phi},\ \Delta\boldsymbol{\Phi} = -\frac{q}{\varepsilon_0},\ q(s,z),$$
$$\Delta = \frac{\partial^2}{\partial s^2} + \frac{1}{s}\frac{\partial}{\partial s} + \frac{\partial^2}{\partial z^2}, \tag{20}$$

$$\vec{B} = \nabla\phi\times\nabla\psi,\ \Delta_\star\psi = \mu_0 s j_\phi,\ j_\phi(s,z),$$
$$\Delta_\star = \frac{\partial^2}{\partial s^2} - \frac{1}{s}\frac{\partial}{\partial s} + \frac{\partial^2}{\partial z^2}, \tag{21}$$

$$\vec{j} = \frac{1}{\mu_0}(\Delta_\star\psi)\nabla\phi, \tag{22}$$

while the toroidal flow is related to the current density by

$$j_\phi = q\upsilon_\phi. \tag{23}$$

With the axisymmetric functions $\rho_0(s,z)$, $\upsilon(s,z)$ the continuity equation (11) is satisfied. The operators Δ and $\Delta_\star$ can be inverted with the aid of Green's functions describing continuous electromagnetic fields that vanish at infinity. The momentum equation (13) then corresponds to two integral equations which have to be solved for the three unknown functions υ, q, $\rho_0(s,z)$.

A dimensionless consideration yields a radius which is of the order of the classical electron radius $r_0 = e^2\mu_0/m$, where e and m are the electron's charge and mass, respectively. Details of the theory and numerical evaluation will be presented in a forthcoming paper where it will also be shown that among all stationary solutions there is one which is stable.

For the large aspect ratio case the external electromagnetic fields $\boldsymbol{\Phi}$ and ψ have been evaluated using Mathematica [6] and are shown in Figs. 1 and 2.

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